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Page 1

D2 - Mathematics

Topic - Non-linear partial differential equation

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eg. Solve $p^2 = z^2(1 - pz)$

Solution : Given that

$$p^2 = z^2(1 - pz)$$

Put $u = x + ay$

Then $p = \frac{dz}{du}$ and $q = a \cdot \frac{dz}{du}$

From the given equation, we have

$$\left(\frac{dz}{du}\right)^2 = z^2 \left(1 - \frac{dz}{du} \cdot a \cdot \frac{dz}{du}\right)$$

$$\Rightarrow \left(\frac{dz}{du}\right)^2 = z^2 \left\{1 - a \left(\frac{dz}{du}\right)^2\right\}$$

$$\Rightarrow \left(\frac{dz}{du}\right)^2 = z^2 - az^2 \left(\frac{dz}{du}\right)^2$$

$$\Rightarrow \left(\frac{dz}{du}\right)^2 + az^2 \left(\frac{dz}{du}\right)^2 = z^2$$

$$\Rightarrow \left(\frac{dz}{du}\right)^2 (1 + az^2) = z^2$$

$$\Rightarrow \left(\frac{dz}{du}\right)^2 = \frac{z^2}{1 + az^2} \Rightarrow \frac{dz}{du} = \frac{z}{\sqrt{1 + az^2}}$$

$$\Rightarrow \frac{\sqrt{1+az^2}}{z} dz = du$$

$$\Rightarrow \frac{1+az^2}{z\sqrt{1+az^2}} dz = du$$

$$\Rightarrow \left(\frac{1}{z\sqrt{1+az^2}} + \frac{az}{\sqrt{1+az^2}} \right) dz = du$$

$$\Rightarrow \int \left(\frac{1}{z\sqrt{1+az^2}} + \frac{az}{\sqrt{1+az^2}} \right) dz = u + C$$

$$\Rightarrow u + C = \int \frac{dz}{z\sqrt{1+az^2}} + a \int \frac{z dz}{\sqrt{1+az^2}} = I_1 + aI_2 \quad (1)$$

For I_1 , put $z = \frac{1}{t}$ so that $-\frac{1}{t^2} dt = dz$

$$I_1 = \int \frac{-\frac{1}{t^2} dt}{\frac{1}{t} \sqrt{1+\frac{a}{t^2}}} = \int \frac{-\frac{1}{t^2} dt}{\frac{1}{t^2} \sqrt{t^2+a}} = -\int \frac{dt}{\sqrt{t^2+a}}$$

$$= -\log(t + \sqrt{t^2+a}) = -\log\left(\frac{1}{z} + \sqrt{\frac{1}{z^2}+a}\right)$$

$$= -\log\left(\frac{1 + \sqrt{1+az^2}}{z}\right) = -\log(1 + \sqrt{1+az^2}) + \log z$$

For I_2 , put $1+az^2 = t$ so that $a \cdot 2z dz = dt$

$$\therefore I_2 = \int \frac{dt}{2a\sqrt{t}} = \frac{1}{2a} \cdot 2\sqrt{t} = \frac{1}{a} \sqrt{1+az^2}$$

Hence from (1) $u + C = -\log(1 + \sqrt{1+az^2}) + \log z + \sqrt{1+az^2}$

$$\Rightarrow \text{ant yfc} = \log \frac{z}{1 + \sqrt{1+az^2}} + \sqrt{1+az^2}$$